Name: Nafeel Entry No.: 1182

INDIAN INSTITUTE OF TECHNOLOGY ROPAR

EE201 - Signals and Systems

First Semester of Academic Year 2023 - 2024

Mid-Semester Examination

Duration: 2 Hours *Maximum Marks: 50* *Date: 20th Sep., 2023*

Instructions: *Calculators are allowed. Calculator exchange is not allowed. Mobile computing devices are not allowed. Specify units properly. Begin each question on a new page. Attempt all questions. No query related to this question paper would be entertained during the exam. In case of any doubt, specify the same in the answer script.*

1. (a) Sketch the following signal [5]

$$x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$$

(b) The sinusoidal signal [5]

$$x(t) = 3cos(200t + \frac{\pi}{6})$$

is passed through a square-law device whose input-output relation is defined by

$$y(t) = x^2(t)$$

Determine the

(i) DC component of $y(t)$,

(ii) Amplitude and fundamental frequency of $y(t)$.

2. (a) Consider an LTI system with input and output related through the equation [5]

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2) d\tau$$

What is the impulse response $h(t)$ of the system?

(b) Determine the response to this system when the input $x(t)$ is as shown below. [5]

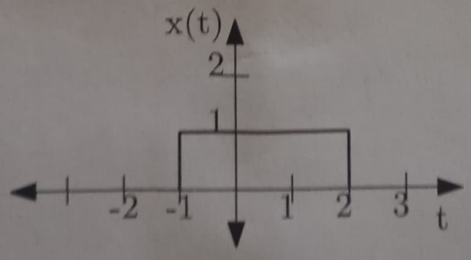

Figure 1: Q. 2(b)

3. (a) Let $x(t)$ be a periodic signal whose Fourier series coefficients are [5]

$$a_k = \begin{cases} 2, & k = 0 \\ j(\frac{1}{2})^{|k|}, & otherwise \end{cases}$$

Answer the following questions with proper justification:

(i) Is $x(t)$ real?

(ii) Is $x(t)$ even?

(b) Let $x(t)$ be a periodic signal with fundamental period T and Fourier series coefficients a_k. Derive the Fourier series coefficients of each of the following signals in terms of a_k: [5]

(i) $x(t-t_0)-x(t+t_0)$

(ii) $x(4t)$

4. (a) Find the Fourier series coefficients for the signal $x[n]y[n]$, where [5]

$$x[n] = cos(\pi n/3)$$

and

$$y[n] = sin(\pi n/6)$$

The signal is periodic with period 12.

(b) Consider two discrete-time periodic signals $x[n]$ and $y[n]$ with period N and their Fourier series coefficients are given as a_k and b_k, respectively. Define a sequence $z[n]$ given as [5]

$$z[n] = \sum_{r=0}^{N-1} x[r]y[n-r]$$

Derive the Fourier series coefficients of $z[n]$ in terms of a_k and b_k.

5. (a) Determine the frequency domain representation of [5]

$$x(t) = \int_{-\infty}^{t} \frac{sin(2\pi\tau)}{\pi\tau} d\tau$$

(b) Using Fourier transform pairs and properties, determine the time domain representation of [5]

$$X(j\omega) = \frac{1}{j\omega(j\omega+2)} - \pi\delta(\omega).$$

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INDIAN INSTITUTE OF TECHNOLOGY ROPAR

EE201 - Signals and Systems

First Semester of Academic Year 2023 - 2024

End-Semester Examination

Duration: 3 Hours *Maximum Marks: 60* *Date: 22 nd Nov., 2023*

Instructions: *Calculators are allowed. Calculator exchange is not allowed. Mobile computing devices are not allowed. Specify units properly. Begin each question on a new page. Attempt all questions. No query related to this question paper would be entertained during the exam. In case of any doubt, specify the same in the answer script.*

1. (a) The raised-cosine pulse $x(t)$ is defined by the following equation. Determine the total energy of the signal. [5]

$$x(t) = \begin{cases} \frac{1}{2}[\cos(\omega t)+1], & -\pi/\omega \le t \le \pi/\omega \\ 0, & \text{otherwise.} \end{cases}$$

(b) A signal is given as $x(t) = e^{-6t}u(t) * \frac{\sin(Wt)}{\pi t}$ [$*$ depicts convolution]. The signal is sampled with sampling interval T_s. [5]

(i) Determine the bound on T_s, which guarantee that there is no aliasing.

(ii) Draw the spectrum of the discrete-time sequence generated when the signal is sampled at twice the Nyquist rate.

2. (a) Sketch the frequency response of the system described by the following impulse response. Characterize the system as lowpass, bandpass, or highpass. [5]

$$h(t) = \delta(t) - 2e^{-2t}u(t)$$

(b) A system is described by the following system function. [5]

$$H(j\omega) = \frac{2+3(j\omega)-3(j\omega)^2}{1+2(j\omega)}$$

Determine the differential equation description for the system.

3. (a) The frequency domain representation of a discrete time signal is given by [5]

$$X(e^{j\Omega}) = \cos(4\Omega)\left[\frac{\sin(\frac{3}{2})\Omega}{\sin(\frac{\Omega}{2})}\right].$$

Determine the corresponding time domain signal.

(b) You are given $x[n] = n(\frac{3}{4})^{|n|} \overset{DTFT}{\longleftrightarrow} X(e^{j\Omega})$. Without evaluating $X(e^{j\Omega})$, find $y[n]$ if [5]

i. $Y(e^{j\Omega}) = e^{-j4\Omega}X(e^{j\Omega})$

ii. $Y(e^{j\Omega}) = \mathrm{Re}\{X(e^{j\Omega})\}$

4. (a) A system has the indicated transfer function $H(s)$. Determine the impulse response, assuming (i) that the system is causal, (ii) that the system is stable. [5]

$$H(s) = \frac{2s^2 + 2s - 2}{s^2 - 1}$$

(b) Determine (i) whether the system described by the following transfer function is both stable and causal and (ii) whether a stable causal inverse exists? [5]

$$H(s) = \frac{(s+1)(s+2)}{(s+1)(s^2 + 2s + 10)}$$

5. (a) A causal system has input $x[n]$ and output $y[n]$. Use the transfer function to determine the impulse response of this system. [5]

$$x[n] = \delta[n] + \frac{1}{4}\delta[n-1] - \frac{1}{8}\delta[n-2]$$

$$y[n] = \delta[n] - \frac{3}{4}\delta[n-1]$$

(b) Use the following clues to determine the signal $x[n]$ and rational z-transform $X(z)$: [5]
$X(z)$ has poles at $z = 1/2$ and $z = -1$, $x[1] = 1$, $x[-1] = 1$, and the ROC includes the point $z = 3/4$.

6. (a) For two continuous-time periodic signals ($x(t)$ and $y(t)$) with period T_0, its periodic convolution is given as $z(t)$. Derive the Fourier series coefficients of $z(t)$ in terms of the Fourier series coefficients of $x(t)$ and $y(t)$. [5]

(b) Determine the time-domain signal $x[n]$ whose discrete-time Fourier Series coefficient is [5]

$$a_k = \sum_{m=-\infty}^{\infty} (-1)^m (\delta[k-2m] - 2\delta[k+3m])$$